with which the data do not agree. It is interesting, however, that in the cases of three of the five metals studied the experimental data agree with the results obtained from the commentators' analytical expression. It appears important, therefore, for a ballistic engineer to analyze the suggested expression and determine its limitations. Perhaps it is limited to hard metals, since the discrepancies were noted in the softest metals studied, gold and aluminum.

More important than the discrepancy cited by Mark and Mirtich is the agreement between the experimental data and the theoretical expression of workers at the Marshall Space Flight Center at Huntsville, Ala.[9] This correlation was found to be true for all metals tested, including gold and aluminum.

The argument of the commentators pertaining to the α/ϵ value of SS 304 is not clear. It is not obvious that the already low α/ϵ value of SS 304 could not be expected to be reduced on particle impact. Quite the contrary is true. In the Fig. 9, we showed that gold, platinum, and chromium plated copper all had lower α/ϵ values after impacting than SS 304 did. The commentators' references to their own work (Ref. 4) shows that the α/ϵ of SS 304 did decrease from 3.86 to 2.0. Contrary to this, they found that the equilibrium temperature of a SS 304 disk remained constant when heated by solar radiation before and after impact. They explained this by the thermal interaction of the disk and the simulated satellite on which it was mounted during the measurement. If such a thermal interaction did take place, then a large portion of their findings are meaningless, since one of the purposes of their work was to determine the equilibrium temperature of disks heated and cooled by thermal radiation only. Although a small but known amount of nonradiant interaction may be accommodated, any interaction that allows the α/ϵ ratio to be reduced by nearly 50% without showing a change in the equilibrium temperature should be investigated and defined.

Contrary to the conclusions of Mark and Mirtich, one of the values of the subject article[1] is that it experimentally substantiates an analytical expression[9] relating changes in optical properties of metals to surface damage. Once it becomes possible to estimate the satellite surface damage caused by the micrometeoroid flux, it should now be possible to calculate the expected change in optical properties with more confidence.

References

[1] Gannon, R. E., Laszlo, T. S., Leigh, C. H., and Wolnik, S. J., "Effect of microparticle impact on the optical properties of metals," AIAA J. **3**, 2096–2103 (1965).

[2] Mirtich, M. J. and Mark, H., "Alteration of surface optical properties by high-speed micron-size particles," NASA SP-55, pp. 473–481 (March 1964).

[3] Mark, H., Goldberg, G., and Mirtich, M. J., "Determination of cratering energy densities for metal targets by means of reflectivity measurements," AIAA J. **2**, 965–966 (1964).

[4] Mark, H., Sommers, R. D., and Mirtich, M. J., "Effect of surface thermal properties of calibrated exposure to micrometeoroid environment," AIAA Paper 65-138 (January 1965).

[5] Leigh, C. H., "Spectral emissivity of metals after damage by particle impact," NASA Marshall Space Flight Center, Contract NAS 8-1642, RAD-TR-62-33 (August 1962).

[6] Laszlo, T. S. and Cannon, R. E., "Spectral emissivity of metals after damage by particle impact," NASA Contract NAS 8-1642, RAD-TR-63-39 (September 1963).

[7] Laszlo, T. S. and Gannon, R. E., "The effect of micrometeoroid impact on the emissivity and absorptivity of metals," Proceedings of the Conference on Materials for Space Vehicle Use, Society of Aircraft Materials and Process Engineers, Seattle, Wash. (November 1963).

[8] Alexander, W. M., McCracken, C. W., Socretan, L., and Berg, O. E., "Review of direct measurements of interplanetary dust from satellites and probes," NASA Goddard Space Flight Center, TN D-1669, pp. 39–60(May 1963).

[9] "Optical properties of satellite materials—the theory of optical and infrared properties of metals," NASA Marshall Space Flight Center, TN D-1523 (1963).

More Results on Solar Influenced Libration Point Motion

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THE influence of the sun on the motion of an artificial satellite moving in the vicinity of the stable earth-moon libration point L_4, was considered in Refs. 1–4. Reference 2 presented a numerical integration of the equations of motion to 2500 days. The results indicated that the motion is oscillatory and has a gross period of approximately 1450 days, with the excursion envelope reaching a maximum amplitude of 160,000 miles. The authors suggested a long-term pulsation for the amplitude of the satellite motion and concluded that the motion was stable. The purpose of this study is to consider the motion for an extended period of time with a view towards an examination of this conclusion. The perturbing effect of the sun is restricted to its gravitational attraction and does not include solar radiation pressure. The earth and the moon are assumed to move about their barycenter in circular orbits. The barycenter, in turn, is assumed to move in a circular orbit about the sun. The earth-moon orbital plane is assumed to be inclined to the ecliptic at a constant angle of 5.15°. Each of the bodies is treated as a mass point. The equations of motion are integrated numerically using the program of Ref. 5. The numerical values for the constants of the problem are identical to those used in Refs. 1 and 2, as are the maximum allowable error and the allowable range of step size. Initial time is taken at inferior conjunction.

Fig. 1 presents the excursion from L_4 for the 5000 day duration of the study. In studying this result, one observes initially a periodicity in the motion envelope of about 1450 days, as noted in Ref. 2. However, the nature of the motion changes after approximately 3700 days. Whereas during the first 3700 days the largest displacement encountered was 167,710 miles, which is approximately two-thirds of the distance between the libration point and the earth or the moon, the magnitude of the displacement for $t > 3700$ days becomes increasingly larger until, at 5000 days, it reaches 52,200,000 miles and the satellite obviously is out of cislunar space and has entered a heliocentric orbit.

The problable explanation for this behavior is the fact that at the "critical" time, the satellite passes at close proximity to the moon, as shown in Fig. 2, which illustrates these passages in the time period 4225 to 4434 days. It is conjectured that the combination of the sun's attractive force and the high satellite velocity during close encounter acts so as to increase the satellite energy sufficiently to escape cislunar space.

It should be pointed out that these results are not believed to be affected grossly by computation errors, since a com-

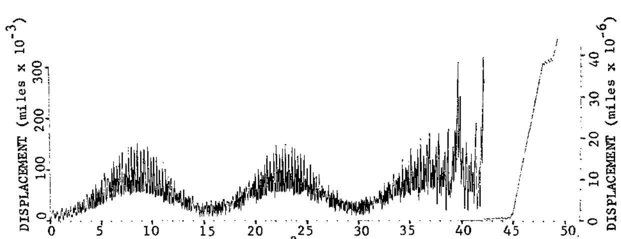

Fig. 1 Excursion of satellite from L_4. (Scale at right refers to $t > 4200$ days.)

Received April 14, 1966

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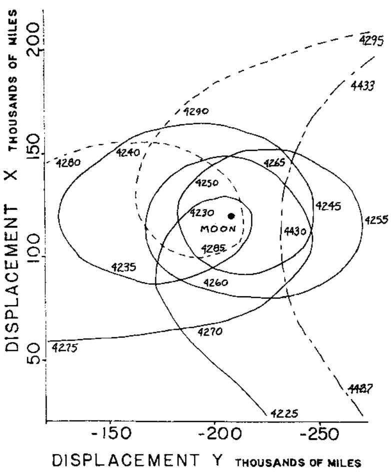

Fig. 2 Close encounter with moon (numbers refer to elapsed days).

parison of the numerical integration with analytical results for a similar two-body problem, for the same number of integration steps, showed a difference of less than 0.0025%.

The results demonstrate that under the sun's gravitational attraction, it is possible for a satellite that is placed initially at rest at a triangular libration point to escape eventually cislunar space and enter a heliocentric orbit following a close encounter with the moon. For the case studied, the motion is bounded for approximately 3700 days, during which time the satellite remains in the vicinity of the libration point. The effects of different initial conditions remain yet to be determined.

References

[1] Tapley, B. D. and Lewallen, J. M., "Solar influence on satellite motion near the stable Earth-moon libration points," AIAA J. **2**, 728–732 (1964).

[2] Tapley, B. D. and Schutz, B. E., "Further results on solar influenced libration point motion," AIAA J. **3**, 1954–1956 (1965).

[3] Schechter, H. B. and Hollis, W. C., "Stability of the trojan points in the four-body problem," The Rand Corporation, Santa Monica, Calif., Memo. MR-3992-PR (September 1964).

[4] De Vries, J. P., "Motion of a particle in the vicinity of a triangular libration point in the Earth-moon system," General Electric Space Sciences Lab., King of Prussia, Pa., Rept. R63S99 (November 1963).

[5] Lastman, G. G., "Solution of N simultaneous first order differential equations by the Adams-Moulton method using a Runge-Kutta starter and partial double precision arithmetic," The Univ. of Texas Computation Center, UTD2-03-046 (D2 UTEX RKAM) (January 1964).

Comment on "Choked Flow: A Generalization of the Concept and Some Experimental Data"

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In the paper[1] the authors make extensive and generous reference to a paper by myself and two co-authors.[2] However they state on p. 2177 that their equations are a general form of the one obtained by Pearson, Holliday, and Smith (PHS), but the point of view adopted and the assumptions made differ from ours. Indeed, "One of the assumptions . . . is the exact opposite of the corresponding assumption of PHS."

Received February 18, 1966.
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Reference to p. 2179 indicates that this opposite assumption is that the authors assume J the momentum to be a minimum whereas we have assumed it to be a maximum. This latter is a simple typographical error in our paper (p. 799). In any case a simple review of the equations will reveal that we only assumed a "stationary" value to the momentum, which actually can only occur for a minimum of the momentum. There is thus no difference in the assumptions used in the two papers. I apologise for the confusion caused by our overlooking the typographical error.

Also on p. 2179, the authors appear to question our assumption that the driving stream is not isentropic and prove that if the driven stream is isentropic, the driving stream also must be. This is true in the region where both streams are flowing parallel and generalized choking occurs. We were concerned in our case with the ejector nozzle where, in the expansion process when the streams are not parallel, substantial losses due to shock waves occur. We therefore were concerned to derive the equations without assuming isentropic flow at all in the driving stream.

References

[1] Hoge, H. J. and Segars, R. A., "Choked flow: a generalisation of the concept and some experimental data," AIAA J. **12**, 2177–2183 (1965).

[2] Pearson, H., Holliday, J. B., and Smith, S. F., "A theory of the cylindrical ejector supersonic propelling nozzle," J. Roy. Aeronaut. Soc. **62**, 746–751 (1958).

Comments on Heat Induced Vibrations of Elastic Beams, Plates, and Shells

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In an article by H. Kraus,[1] which was concerned with thermally induced vibrations of thin elastic nonshallow spherical shells, the author demonstrates, as have previous authors,[2–4] that there exists the possibility of thermally induced vibrations of thin elastic beams, plates, and shells. It should be noted, however, that, since the introduction of this problem by Boley[2] in 1956, there has been no experimental evidence to support any of this analytical work. Lyons,[5] in fact, has shown that thermally induced transverse vibrations of thin elastic plates are not possible under the present conception of heat input. The present conceptions are that heat is introduced into the elastic system by prescribing on the surface of the beam, plate, or shell either an instantaneous heat flux or a temperature.

The diffusion of heat in time across the thickness of the elastic member is a necessary consideration in both of the previously considered heat inputs. Therefore, when this diffusion is eliminated in normal reduction to the one-dimensional beam problem, or the two-dimensional plate or shell problem, it is evident how these thin elastic members theoretically could exhibit vibrations because of their instantaneous surface heat inputs. This could occur since this spatial reduction imposes an infinite velocity of heat diffusion across the thickness of the elastic members.

There remains only one possible method to induce thermally vibrations in an elastic thin beam, plate, or shell. This method involves the direct instantaneous supplying of heat energy to each material element of the structural member, without depending upon thermal diffusion to transfer it.

Received April 4, 1966. This work was supported by the U. S. Atomic Energy Commission.
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